EDITORIAL

The Last Conundrum

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[This editorial preceded the article "Zero-Pressure-Gradient Turbulent Boundary Layer," by W. K. George and L. Castillo, which appeared *in Applied Mechanics Reviews*, vol. 50, no. 12, part 1, December 1997.]

Turbulence is the last great unsolved problem of classical physics. Or so it goes for a quote variously attributed to one of the great modern physicists Albert Einstein, Richard Feynman, Werner Heisenberg and Arnold Sommerfeld. But in fact the closest sentiments to this quote that could be traced are due to the classical physicist Horace Lamb who actually wrote starting with the second edition of his celebrated book Hydrodynamics (1895) under the heading of Turbulent Motion: "It remains to call attention to the chief outstanding difficulty of our subject." A more humorous fable, also attributed to several of the great ones, goes as follows. As he lay dying the modern physicist asked God two questions: Why relativity (or quantum mechanics, depending on who is departing), and why turbulence? "I really think," said the famed physicist, "He may have an answer to the first question." No one knows how to obtain stochastic solutions to the well-posed set of partial differential equations that governs turbulent flows. Averaging those nonlinear equations to obtain statistical (nonstochastic) quantities always leads to more unknowns than equations, and ad hoc modeling is then necessary to close the problem. So, except for a rare few limiting cases, first-principle analytical solutions to the turbulence conundrum are not possible. In the words of John Lumley, turbulence is a difficult problem that is unlikely to suddenly succumb to our efforts. We should not await sudden breakthroughs and miraculous solutions, but rather keep at it slowly building one small brick at a time.

Two of the greatest achievements of turbulence research are the Kolmogorov's universal equilibrium theory and the universal logarithmic law of the wall. In fact, there is a direct analogy between the two high-Reynolds-number asymptotes, one being concerned with a cascade of energy and an inertial subrange in the frequency domain and the other with a hierarchy of eddies and an inertial sublayer in the physical space. The overall flow dynamics in both the energy spectrum subrange and the wall-bounded flow sublayer is independent of viscosity. Dimensional reasoning, similarity and asymptotic analysis are the tools of choice to derive analytical expressions without actually solving the intractable governing equations.

One of the fundamental tenets of boundary layer research is the idea that any statistical turbulence quantity (mean, rms, Reynolds stress, etc.) measured at different facilities and at different Reynolds numbers will collapse to a single universal profile when non-dimensionalized using the proper length and velocity scales (different scales are used near the wall and away from it). This is termed self-similarity or self-preservation and allows convenient extrapolation from the low-Reynolds-number laboratory experiments to the much higher Reynolds number situations encountered in typical field applications. The universal logarithmic profile mentioned above describes the mean streamwise velocity in the overlap region between the inner and outer layers of any wall-bounded flow, and is the best known result of the stated classical idea.

The log-law has been derived independently by Ludwig Prandtl and G. I. Taylor using mixing length arguments, by Theodore von Kármán using dimensional reasoning, and by Clark B. Millikan using asymptotic analysis. Those names belong of course to the revered giants of our field. Questioning the fundamental tenet or its derivatives is, therefore, tantamount to heresy. But the questions and doubts linger as evidenced from the work of Simpson (1970), Malkus (1979), Barenblatt (1979), Long (1981), Willmarth (1989), George (1992), Bradshaw (1993), Sreenivasan (1993), Smits (1994), among others, who at different times challenged various aspects of this law. And those are only the ones who, with varying degrees of difficulty, could get their work published. There is strong suspicion, among the sacrilegists at least, that Reynolds number effects persist indefinitely for both mean velocity and, more pronounceable, higher-order statistics, and hence that true self-preservation is never achieved in a growing boundary layer. In fairness to the high priests, their logarithmic law was always intended to be a *very* high-Reynolds-number asymptote. These issues and the cited references could be found in greater details in the survey by Gad-el-Hak and Bandyopadhyay (*Applied Mechanics Reviews* **47**, no. 8, pp. 307–365, 1994).

If in fact the log-law is fallible, the implications are far reaching. Resolution of the full equations, via direct numerical simulations, at all but the most modest values of Reynolds number is beyond the reach of current or near-future computer capabilities. Modeling will, therefore, continue to play a vital role in the computations of practical flows using the Reynolds-averaged Navier-Stokes equations. Flow modelers, in attempting to provide concrete information for the designers of, say, ships, submarines and aircraft, heavily rely on similarity principles in order to model the turbulence quantities and circumvent the well known closure problem. Since practically all turbulence models are calibrated to reproduce the law of the wall in simple flows, failure of this universal relation virtually guarantees that Reynolds-averaged turbulence models would fail too. Finally, developers of flow control devices to reduce drag, enhance lift, etc., often have to extrapolate the widely available low-speed (or more precisely low-Reynolds-number) results to high-speed flows of practical interest where no data are available. Such extrapolation is not possible if the difficult-to-quantify Reynolds number effects persist indefinitely. For both scientists and engineers the message is essentially back to the drawing board!

In a community of conformists, the heretics never have it easy, of course. The peer review system, while essential for weeding out the charlatans, the misguided and the fools, is somewhat biased against unorthodox ideas. Nevertheless, the latest two papers to question the infallibility of the log-law are the article by George and Castillo that follows this editorial and the one by Barenblatt, Chorin and Prostokishin, also published in *Applied Mechanics Reviews* (vol. 50, no. 7, pp. 413–429, 1997). The two teams tackle the same problem quite differently and independently. Both papers offer concrete alternatives to the Reynolds-number-independent law of the wall. Barenblatt et al. use scaling laws that invoke a zero-viscosity asymptote, while George and Castillo introduce new tools they term asymptotic invariance principle (AIP) and near asymptotic, which result in a new law of the wall with explicit Reynolds number dependence. George and Castillo's new 'law' is deduced from first principles and fits existing mean-velocity data better. Significantly, the same methodology advanced by William George applies to higher-order statistics as well.

When the log-law and its consequences are challenged, the usual immediate reaction is to doubt the credentials of the blasphemer. Something is wrong with her model, with his experiment, with her numerical scheme, or with an endless list of potential pitfalls. These are all genuine concerns that turn out to be valid most of the time, but paranoiacs have enemies too! There is also the persistent albeit misguided argument that the log-law fits the data well enough for engineering applications. If it isn't broke, don't fix it! This pragmatism is of course simultaneously the curse and the blessing of science conducted by engineers. Moreover, while the errors involved in attempting to fit the log-law to existing mean-velocity data are quite tolerable considering our inability to accurately measure the friction velocity (the velocity scale necessary to collapse the plots), the corresponding errors for higher-order statistics are egregious. The entire enterprise is not unlike the sixteenth century debate over the Ptolemaic view of the heavens and the Copernicus' model seeking to replace it. The former theory served navigators well for over 1400 years. The Copernican theory made only small corrections but radically changed man's view of his universe. And it was a masterpiece in terms of its economy of postulates and assumptions, a necessary condition for theoretical elegance.

In the paper that follows, the readers of *AMR* are treated to what is perhaps the most rigorous challenge to date to the law of the wall. The article is a review but not in the traditional sense: though centered around developing a new theory that offers a viable alternative to the classical logarithmic velocity profile and its consequences, the novel theory is validated using an abundant of experimental data available in the open literature. In that sense then the paper represents a complete review of its subject matter. The theory presented for the zero-pressure-gradient boundary layer is derivable from first principles and is extendible to other wall-bounded flows including channel and pipe flows, boundary layers with pressure-gradient, and wall jets. It is hoped that the publication of the following clearly iconoclastic article will spur a new way of looking at wall-bounded flows and of challenging the status quo. One should of course ask the usual barrage of questions that only a vibrant collection of skeptical neurons could muster. Is the theory simple, elegant and self-consistent? Does the model provide a better fit to the data? Does it explain previous contradictions? Is the theory based on a minimum number of assumptions? Is it extendible to more complex situations? Are the results asymptotically correct? Is the logic sound? Is the mathematics free of errors? .... A fitting end to this editorial is to once again quote John Lumley, my (elder) academic sibling and the doctoral thesis advisor of William George. ".... A theory that does all that in an effortless way is often called elegant. Tomorrow, it may be wrong. Even so, it deserves to be regarded as one of the better things of which man is capable."